DOCUMENT RESUME

ED 350 040 JC 920 492

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TITLE The Use of Computers in the Math Classroom.
PUB DATE 28 Jan 92
NOTE 51p.; Paper presented at the Annual Winter Institute on Community College Effectiveness and Student Success (1st, Jacksonville, FL, January 26-28, 1992).
PUB TYPE Reports - Descriptive (141) -- Tests/Evaluation Instruments (160) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Algebra; Calculus; College Faculty; *Community Colleges; Computer Assisted Instruction; Computer Literacy; *Computer Oriented Programs; Computer Simulation; Computer Software; *Computer Uses in Education; Differential Equations; *Faculty Development; Instructional Materials; *Mathematics Instruction; Mathematics Materials; Microcomputers; Statistics; Summer Programs; Two Year Colleges
IDENTIFIERS Oakland Community College MI

ABSTRACT

In an effort to increase faculty use and knowledge of computers, Oakland Community College (OCC), in Michigan, developed a Summer Technology Institute (STI), and a Computer Technology Grants (CTG) project beginning in 1989. The STI involved 3-day forums during summers 1989, 1990, and 1991 to expose faculty to hardware and software applications. The CTG project, initiated in academic year 1990-91, offered 25 microcomputers to faculty on a competitive applications basis. Faculty who were awarded computers, as well as printers and $3,000 for additional hardware and software, were asked to submit written progress reports within 1 year, or to present their work at the STI. After an introduction by George Keith, this report provides the following short papers written by CTG grant recipients: (1) "The Use of Computer in Statistics and Algebra," by Barbara Blass, which reviews a three-semester project to develop a computer assisted course, centered on statistical worksheets and solving algebraic equations using quadratic function graphs; (2) "The Use of Computers in Calculus I," by Gladys Rockind, which describes a program to help students visualize the ideas of limits, tangent lines, derivatives, maximums, minimums, integrals, areas, and volumes and to see the relationships among these areas of calculus; and (3) "The Use of Computers in Calculus IV and Differential Equations," by Keith Shuert, which describes the use of computers to display and rotate multivariate surfaces, to graph the fields and solution curves of differential equations, and to conduct numerical simulations. The bulk of the report consists of sample materials and worksheets from the course taught by Blass. (PAA)

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THE USE OF COMPUTERS IN THE MATH CLASSROOM

by

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I. TECHNOLOGY GRANTS FOR FACULTY

...Dr. George Keith

In 1988 administration and faculty leadership recognized the need to provide additional stimulus for academic computing at Oakland Community College. While a core of faculty were well advanced in their use of computer technology, the vast majority had little or no exposure to it. The problem was multi-faceted: how could we support that interest once it had developed; how could we create a development strategy to encompass an increasingly wider circle of faculty and staff in computing applications.

OCC formed the College Information and Instructional Technology Advisory Group(CIITAG) to address these issues. Composed of representative computer-interested faculty from each of our campuses, representatives from the Central Office area of Information and Telecommunications System, and representative campus administrators, the CIITAG developed two major projects as the initial means for expanding faculty computer awareness and use. The first project was a Summer Technology Institute, the second was a Technology Grants program.

The Summer Technology Institute, offered over a three-day period in August 1989 at our Highland Lakes Campus, provided an information forum to expose faculty to hardware and software applications. Representatives from Apple and IBM were present with equipment to display and demonstrate. The major focus, however, was on applications, employing select OCC faculty to present their software use or development work. IBM Consulting Scholar Rudy Gentry was the keynote speaker and presenter. In 1990 the Summer Institute featured IBM Consulting Scholar Edna Coffin and in 1991 James Noblitt. The 1990 and 1991 Institutes expanded on the software and hardware presentations of the preceding year, with increasing numbers of faculty from our campuses demonstrating classroom and course management applications. Both years offered "hands-on" experiences, for example, creating a gradebook using adjunct instructors as well and proved to be popular with them. Participation has steadily increased over the past three years; in the first year 45 participants registered, 70 signed up in 1990, and close to 100 participated in 1991.

The summer institutes have proven to be a successful means of creating an awareness of, and an interest in, academic computing and its concomitant technologies. What the CIITAG also set out to do was provide physical support for these initiatives and did so by creating the Computer Technology Grants project. We recognized early-on that interest and enthusiasm would soon wane if faculty did not have reasonable access to equipment and software. While microcomputer labs exist at all of our campuses, scheduling

and student use prohibit most faculty from using the equipment there because it isn't available when faculty may wish to use it, or the labs are not conveniently located near their classrooms or offices. Faculty as novice users in computer applications often are not comfortable trying to develop their skills in the public arena. Other faculty require extended use of equipment to benefit.

The Computer Technology Grants program, begun in academic year 1990-91, offered 25 microcomputers to faculty on a competitive applications basis. Application requirements stressed reasonable, practical classroom uses and student support service uses. A committee of readers was commissioned for a blind reading, evaluating, and ranking of the submitted proposals. Individual grants for the first year were IBM PS/2 Model 30's and printers, plus software, not to exceed $3000 as a total individual package of hardware and software. Faculty awarded computers were asked to submit in writing a progress report within one year or present their work at the succeeding Summer Institute.

The 1990-91 competition resulted in 53 faculty applications for the 25 computers. The awards were made in January of 1991. The second year of competition is currently under way for another 25 micros to be awarded in January of 1992. This year's competition retains the same individual dollar total, but those awarded may select between Apple or IBM computers.

In sum, the original objectives of increasing computer awareness, literary, and use among faculty have been successfully fulfilled. We will continue to use the institute concept and the grants concept as stepping stones. For example, we are now talking about targeting discipline clusters for institutes and for developing more sophisticated, selective grants for higher level computer projects, along with our established grants program.

The three presentations which follow result in part from the computer technology grants program. Dr. Shuert, Ms. Blass, and Ms. Rockind were award recipients. I say "in part" because their work with computers was in gestation prior to the grants. The grants, however, facilitated their work by bringing the computer resource directly to them.

II. THE USE OF COMPUTER IN STATISTICS AND ALGEBRA
...Barbara Blass

Eight years ago at the 1984 National Council of Teachers of Mathematics Conference, the focus was on the most critical problem facing math education: the need for curricular and instructional methods reflecting the influence of computing on mathematics. In 1992, the problem still faces educators. We need to shift the emphasis from manipulative skills to concepts, problem solving, and critical thinking. Teacher presentations must involve student-directed learning that exploits technology to solve problems and explore concepts. The typical classroom is a predictable pattern of "show & tell & drill", reinforced by textbooks. The instructor describes the procedure and its rationale, complete with a series of illustrative examples, and then assigns long sets of practice exercises. To encourage student exploration, we must provide new situations both in and out of the classroom.

To illustrate the latter, I offer my students computer worksheets for statistics. Almost all students registering for Elementary Statistics will be required to take an advanced statistics course at their transfer institution. After researching the three leading transfer institutions for OCC business majors, I discovered that all require a computer component. Because it was possible for a student to never have hands-on computer experience before leaving OCC, I decided to experiment with assignments completed outside of class. When reviewing the many software packages available, I kept an eye to what I hoped to accomplish. The package had to be very user-friendly, yet address all topics covered by the course, allowing me to write worksheets to encourage the student to think beyond those problems in the text. It had to place the mathematical decision making in the hands of the student.

After selecting STATDISK, I planned a three semester implementation. During the first, a pilot group of 20 would complete assignments on a voluntary basis, keeping journals of difficulties they had with the computer or the worksheets. At the end, they completed critiques allowing me to adjust for the next semester. Critiques addressed the amount of time needed to complete assignments on the computer as well as outside of the lab, friendliness of STATDISK, clarity of directions, and benefit of STATDISK to one's understanding of textbook homework questions. Also students were asked for their suggestions as to how STATDISK should be incorporated into the class in the future. Each semester the program was expanded until it was offered to all in the class. Few chose not to participate.

To introduce the students to the computer, times were set aside outside of class for groups to meet in the computer room and go through the first worksheet problem together. Each of the five worksheets is accompanied by a "Helpful Hints" direction sheet. Students were instructed to work alone on the computer but encouraged to discuss their findings. Many problems instructed the students to have th computer generate a data set and analyze an outcome; collaborative learning would be appropriate. The same computer package was used in class for homework problems and often students would relate situations from their lab which led to lively discussions.

The three semester implementation proved to be a learning process for me as well as the students. I found that in the computer lab, students were active learners, thinking for themselves. Realistic situations would motivate students, and they became more flexible and greater risk-takers in problem-solving situations. The worksheets increased student abilities to express mathematical ideas in writing. Also discussions between students increased their verbal facility with statistics.

Statistics worksheets provided students an opportunity to explore mathematics outside the classroom. In algebra classes at all levels, we have many chances to involve students with simple graphing software or graphing calculators. When selecting software for classroom exploration, I look for programs that the student can immediately use such as TRUE BASIC or CactusPlot. Students take turns at the computer as the class watches the projection on the overhead.

Using the graph of a quadratic function allows for an in-depth discussion of x intercepts. After students learn graphing and factoring trinomials in elementary algebra, I show them the graph of $y = ax^2 + bx + c$. Although they don't know how to graph a parabola, they immediately deduce that different types of equations have different graphs and that x intercepts might occur on graphs other than lines. I am always impressed by the insightful questions raised at this point concerning y intercepts, symmetry, power functions, and finding zeros of higher order polynomials. In more advanced algebra classes, the graphs are quick ways to explore the discriminant.

A second demonstration in algebra is that of solving systems of equations using the graphs. If the student wishes to solve for the points of intersection of $y = x^2$ and $y = 2^x$ (or any similar problem of the form $y = x^p$ and $y = p^x$ where p is a positive integer), he can graph them both using the computer or a graphing calculator. Then listing a table of values or using the calculator trace, he can zoom in on the solutions. Using CactusPlot affords the student another

opportunity. He can graph $2^x - x^2 = 0$ and using $y = 0$, he can let the computer find the x intercepts.

Students often view solving polynomial inequalities and rational inequalities by the use of the number line and signs(cut point method, sign chart, etc.) as a mechanical method to be memorized. By using the graph of the corresponding equation, students understand that the cut points are merely zeros or asymptotes separating regions where the graph falls above or below the x axis. It becomes a visual problem and not a robotic exercise.

III. THE USE OF COMPUTERS IN CALCULUS I
...Gladys Rockind

I have for a long time wanted my students to see the beauty in mathematics in general and in calculus in particular, and to get excited by the relationships between and among the various operations used and the topics studied. To that end I had been for many years using some pencil and paper "discovery-type" lessons which the students typically did before we covered a lesson in class in which I hoped they would be able to develop some of the concepts on their own and get a feel for what would be covered in class. But I still felt there was more that should be happening in this area.

After attending numerous conferences, reading many articles and getting an opportunity through the computer grant to have a computer available to test out the software I had been hearing and reading about(as well as other available software), I decided that there was indeed more that could be done and that I could do it with a series of computer labs and classroom demonstrations on the computer. The labs would be done outside of class and would relate to and reinforce the concepts we were working on in class as well as provide visual input to the subject. The classroom demonstrations were used mainly to provide visual input and tie the various pieces of the calculus puzzle together.

After reviewing various programs I decided on TrueBasic Calculus for the labs and this and Sensie Calculus for the demonstrations. (The academic lab on our campus is a Macintosh lab, so I used the Macintosh version.) I chose the TrueBasic Calculus because of its ease of use and also because it requires the students to think when they enter a function. (They need to enter an interval over which they want the computer to draw the graph and then need to change the y-interval if the one the computer chose is not appropriate. This provided many opportunities to discuss domains and ranges and the places one has to beware of when using graphing tools.) In addition, it provided a table of values with any steps the student chose, and could draw a graph and its first and second derivative easily and quickly on the same axes. (It also had a limit menu, which I had the students use to check their work. I preferred they explore limits with the regular graphing menu.) The Sensei Calculus is a nice all around program with wonderful items to use for classroom demonstrations. I made several copies of this available in the lab for those that wanted to look at the demonstrations again.

My goal was to get the students to visualize the ideas of limits, tangent lines, derivatives, maximums, minimums, integrals, areas, and volumes and to see the relationships between and among these areas of calculus. Although the

labs were often tedious and time-consuming(if the students are to be believed). I felt that the students did have an idea of continuity and what it is; what types of functions are continuous and what types are not; when a function has a limit, and when it does not; what it means to look at limits at infinity; and how a derivative is related to a function and what it was (something I have never thought before that my students knew). When we discussed maximums and minimums, I truly felt they could picture the graphs and the horizontal tangent lines and the relating rule of setting the derivative equal to zero. And if I felt they weren't picturing it as I planned, we could always go to the computer, produce the picture (and the picture of the derivative if we wanted) and see what was happening. I also thought that integrals and areas under the curve were much clearer than ever before. (Both programs allowed us to draw a curve and change the number of rectangles and look at the actual area and the area enclosed by the rectangles.)

From the point of view of accomplishing my goals I was very pleased. I keep reading, looking and thinking about what I can do differently, what I can add, or what I can delete in order to make calculus make more sense, and be more exciting and more meaningful. My major disappointment was that I felt more students who weren't inclined to work and to think, dropped out earlier than usual rather than stay around and see if this new material would help.

IV. THE USE OF COMPUTERS IN CALCULUS IV AND DIFFERENTIAL EQUATIONS ... Dr. Keith Shuert

One way that computers can increase instructional effectiveness in the teaching of mathematics is to capitalize on the computer's ability to graph surfaces and curves that the teacher and the students alike find impossible to visualize, much less draw.

This is certainly true of many of the multivariate surfaces discussed in Calculus IV. As examples consider the following:

1. The surface $z = x^3y^2$ has tangent planes at each point along both the x-axis and the y-axis.

2. The surface $z = y^2 - 2xy + x^2$ has an absolute minimum at each point on the line $y = x$.

3. The surface $z = 2y^2x - yx^2 + 4xy$ has three saddle points and one relative minimum point.

4. The surface $z = e^x \sin y$ has no relative maximum points, no relative minimum points, and no saddle points.

Even though neither the teacher nor his students can imagine these surfaces, the computer can not only display them, but can rotate them for viewing from any angle or perspective.

Consider the problem of showing that the surfaces $z = \sqrt{16 - x^2 - y^2}$ and $z = \sqrt{x^2 + y^2}$ intersect at $(2,2,2\sqrt{2})$, and have tangent planes that are perpendicular at the point. A computer can display these intersecting surfaces simultaneously and the points of intersection can be easily seen.

In Differential Equations the computer is very effective in explaining the nature of differential equations and their solutions by graphing their direction fields and various solution curves. This is impossible to do without a computer. The graph of the direction field can then be used to teach concepts which would not be possible to teach without the computer images:

1. It can be used to show how to determine whether a differential equation has a solution through a given point.

2. It can give an approximation to a numerical value of the solution of an initial-value problem.

3. It can also be used to illustrate that each point in the plane gives rise to a different solution.

4. The direction field can illustrate the "flow pattern" for the family of solution curves of given differential equation.

5. The direction field can be used to establish the existence of a first-order differential equation that cannot be solved by standard techniques.

6. These graphs can draw attention to particular characteristics of solutions such as asymptotic behavior and sensitivity of long range patterns to changes in problem parameters.

The computer is most effective in the classroom when it is used to do numerical simulation. In Differential Equations one can investigate the "predator-prey" model of Volterra and Lotka which models the evolution of two competing species, say foxes and rabbits, and how they influence each other. First you mathematically express the initial values and the transition rules that describe how the model evolves from one generation to the next. Then the computer will plot, both numerically and graphically, the future behavior of the two populations. These displays allow you to answer the following kinds of questions about the model:

1. Will the rabbits outgrow the foxes?

2. Will the foxes decimate the rabbit population and then die out themselves due to starvation?

3. Will the rabbit population decrease as a result of predation, but as a result cause more foxes to starve, thus allowing the rabbits to rebound.

4. In this latter case, is there some final stable state in which the death rate of each species exactly balances the birth rate, or do the two populations oscillate indefinitely?

It would be impossible to answer these questions effectively without the use of a computer.

THE USE OF COMPUTERS IN THE MATH CLASSROOM

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Winter Institute on Community College
Effectiveness and Student Success

January 28, 1992

CACTUSPLOT
Classroom Explorations

I. STATISTICS

M = 0: S = 1: Y = FNN(X) Graphs the standard normal curve.
x:[-4,4] y:Auto
Use the integrate option to display areas under the curve and find probabilities.

II. INTERSECTION OF CURVES

A. $y = x^2$
$y = 2^x$
x:[-3,5] y:Auto

Use a table of values on $y = 2^x - x^2$ along with the graph to determine where y = 0.

B. $y = x^3 - 2x$
$y = 2\cos(x)$
x:[-5,5] y:[-5,5]

Examine the graphs in quadrant 2 and zoom into x:[-1.5,-1] y:Auto to discuss their lack of intersection. Graph $y = 2\cos(x) - (x^3 - 2x)$ on x:[-2,0] y:Auto.

III. CUT POINT METHOD

Graph each function on the given viewport in order to solve the inequality.

$y = x^2 - x - 6$ x:[-7,7] y:[-15,15] Solve $x^2 - x - 6 < 0$

$y = \frac{x^2 - 4}{x - 3}$ x:[-7,7] y:[-15,15] Solve $\frac{x^2 - 4}{x - 3} \geq 0$

$y = (x-2)(x+1)(x+4)$ x:[-5,3] y:[-15,15] Solve $(x-2)(x+1)(x+4) \geq 0$

IV. RATIONAL FUNCTIONS

A. Graph $f(x) = \frac{2x^4 + 7x^3 + 7x^2 + 2x}{x^3 - x + 50}$ x:[-7,7] y:[-15,15]

Discuss the graph's flat behavior on $-3 \leq x \leq 2$. Zoom in on the graph using x:[-2.5,1.5] y:[-.3,.5].

B. Graph $f(x) = \frac{2x^3 + 7x^2 - 4}{x^2 + 2x - 3}$ x:[-7,7] y:[-15,15]

Discuss the graph's behavior near its oblique asymptote. Zoom in on the graph using x:[-3,1] y:[-8,5]

C. Graph $f(x) = \frac{x^2 - 2x + 1}{x^2}$ or $f(x) = \frac{x^3 - 1}{x^3}$ x:[-7,7] y:[-15,15]

Observe and discuss that both graphs cross their horizontal asymptotes.

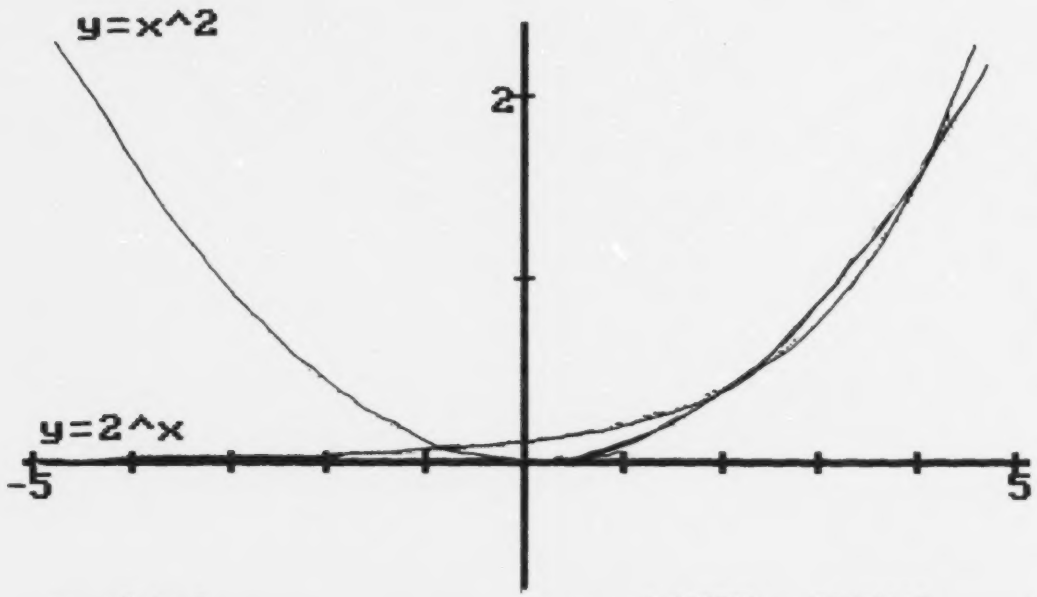
V. PIECEWISE DEFINED FUNCTIONS

$$f(x) = \begin{cases} x^2 + 2x + 5 & \text{if } x < -2 \\ x^3 & \text{if } -2 \leq x \leq 2 \\ 2x - 5 & \text{if } x > 2 \end{cases}$$

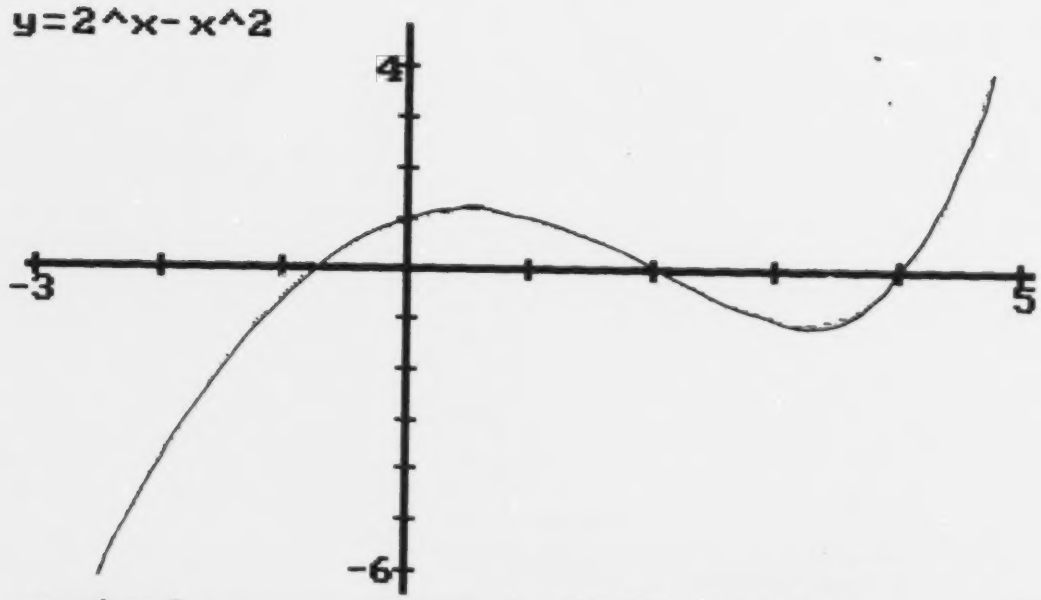
Graph the functions $y = x^2 + 2x + 5$, $y = x^3$, $y = 2x - 5$. Then place vertical lines at $x = -2$ and $x = 2$ using the "write" option. Show the "pasting" of the graphs based on the domain. Encourage students to change the domains and paste view piecewise defined functions.

VI. SEQUENCES

To graph the sequence $a_n = (1/2)^{n-1}$, use a table of values for x from 0 to 10 and the plot option on the command $y = (1/2 \wedge (x - 1)) * FNI(x)$. Students can discuss the limit concept.



x: -5 to 5 y: -6.637755 to 23.34146



x: -3 to 5 y: -6.295993 to 4.605645

```
***TABLE OF VALUES***

 1:   Y=2^X-X^2

X                 Y

 0                 1
 .25               1.126707
 .5                1.164214
 .75               1.119293
 1                 1
 1.25              .815914
 1.5               .578427
 1.75              .301086
 2                 0
 2.25              -.305672
 2.5               -.593146


 1:   Y=2^X-X^2

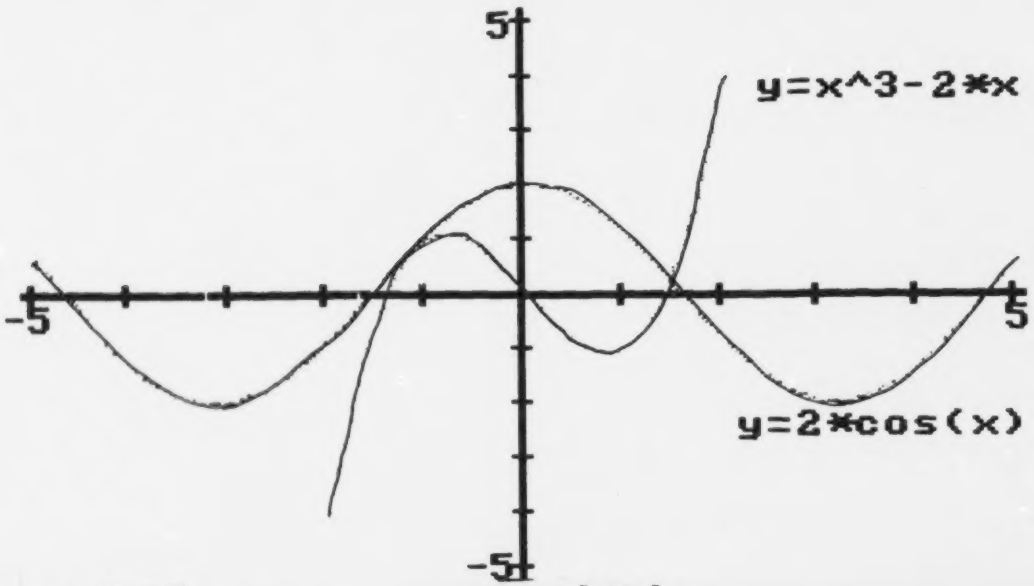
X                 Y

-1.5               -1.896447
-1.35              -1.430208
-1.2               -1.004725
-1.05              -.619532
-.9                -.274113
-.75                .032104
-.6                 .299754
-.45                .529543
-.3                 .722252
-.15                .87875
 0                  1

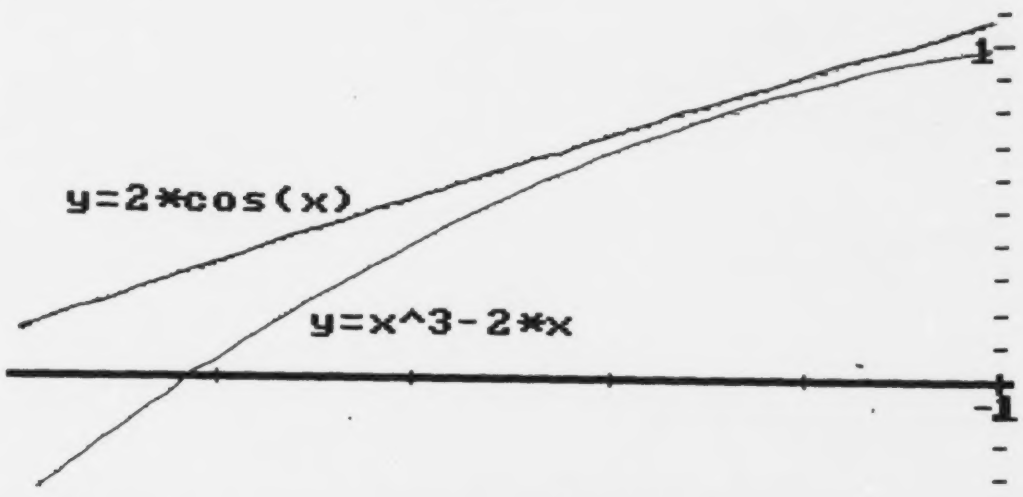

  1:   Y=2^X-X^2

 X                 Y

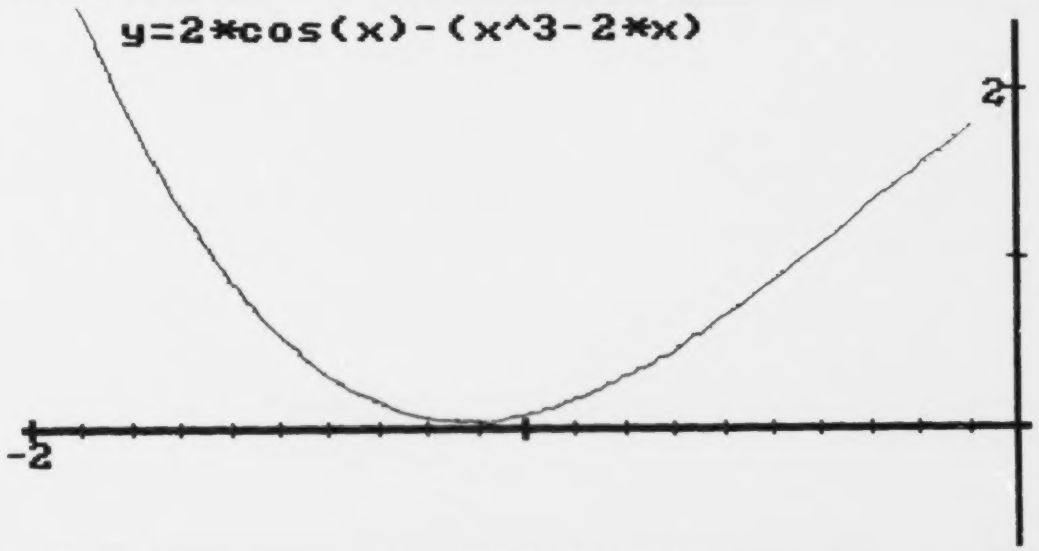
  3.5               -.936292
  3.6               -.834267
  3.7               -.693962
  3.8               -.511191
  3.9               -.281472
  4                  0
  4.1                .338375
  4.2                .739174
  4.3                1.208311
  4.4                1.752127
  4.5                2.377417
```



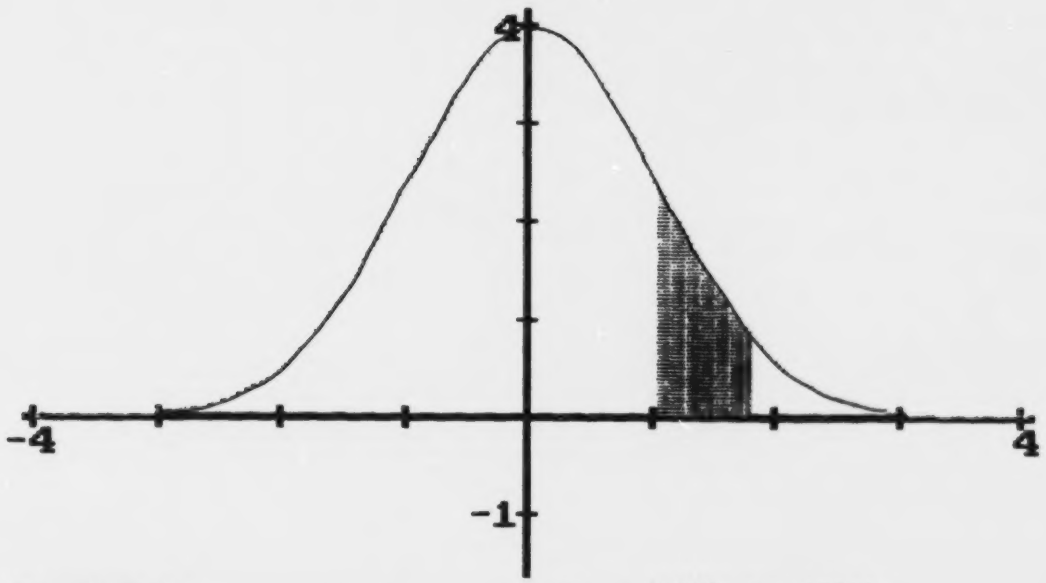
x: -5 to 5 y: -5 to 5



x: -1.5 to -1 y: -.370765 to 1.26208

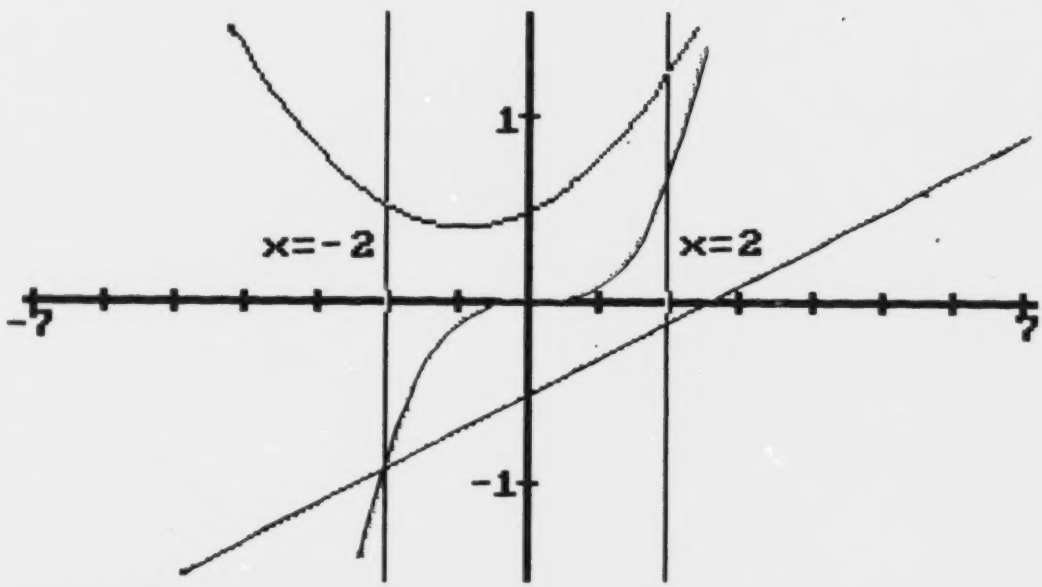


4 x: -2 to 0 y: -.700797 to 2.56208



x: -4 to 4 y: -.155287 to .40527

```
INTEGRAL OF M=0; S=1; Y=FNN(X)
FROM  1  TO  1.75  IS  .118596
```



x: -7 to 7 y: -15 to 15

```
y=x^2+2*x+5    if  x<-2
y=x^3          if  -2<=x<=2
y=2*x-5        if  x>2
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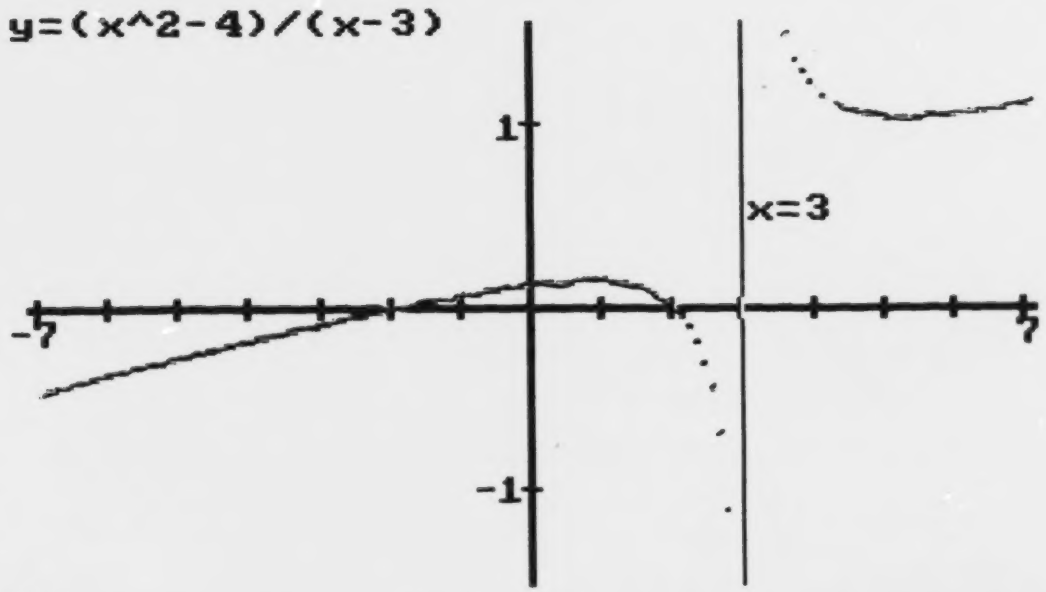
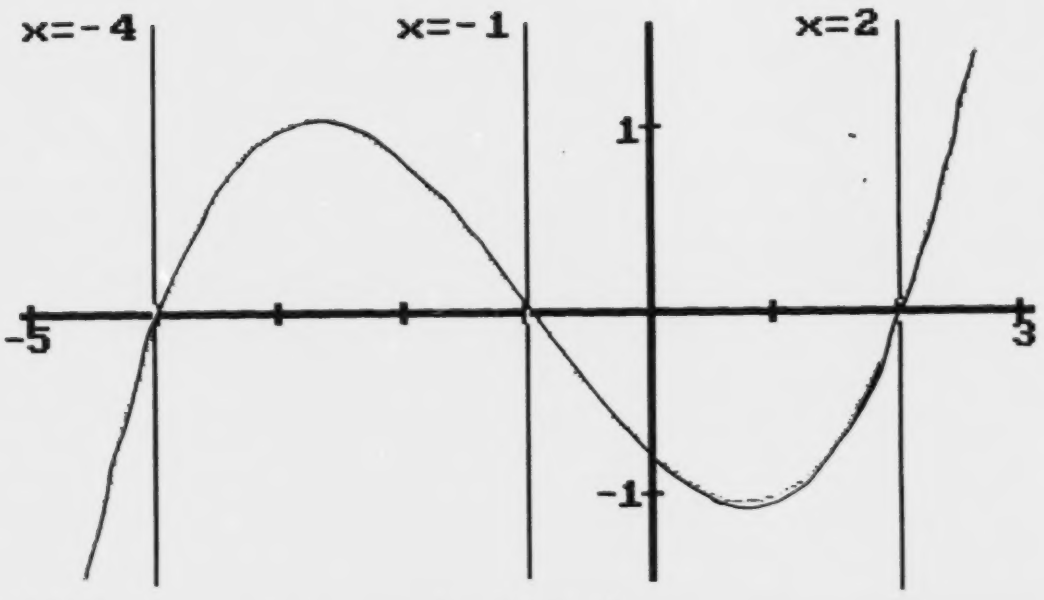


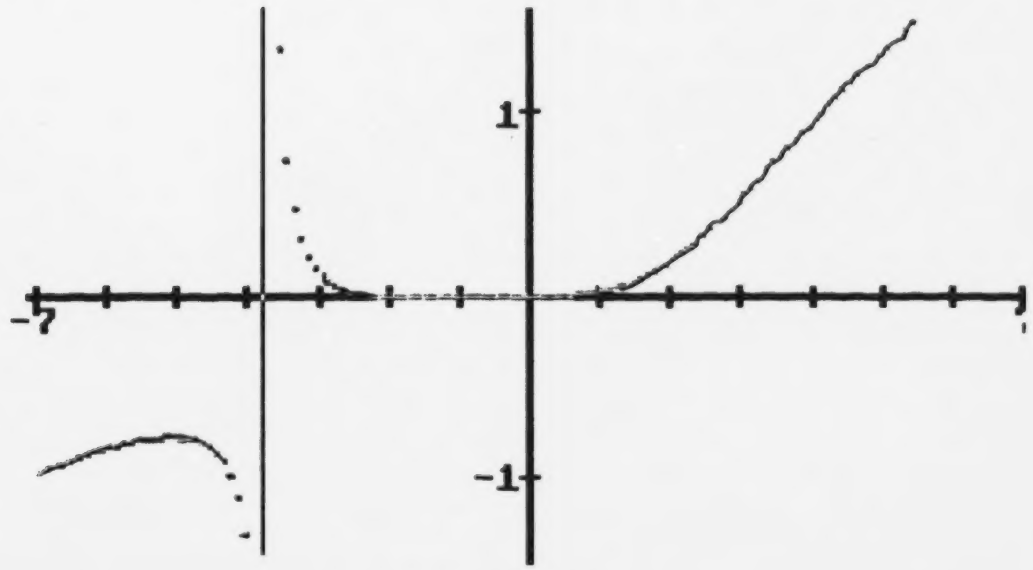
x: -7 to 7 y: -15 to 15



x: -5 to 3 y: -15 to 15

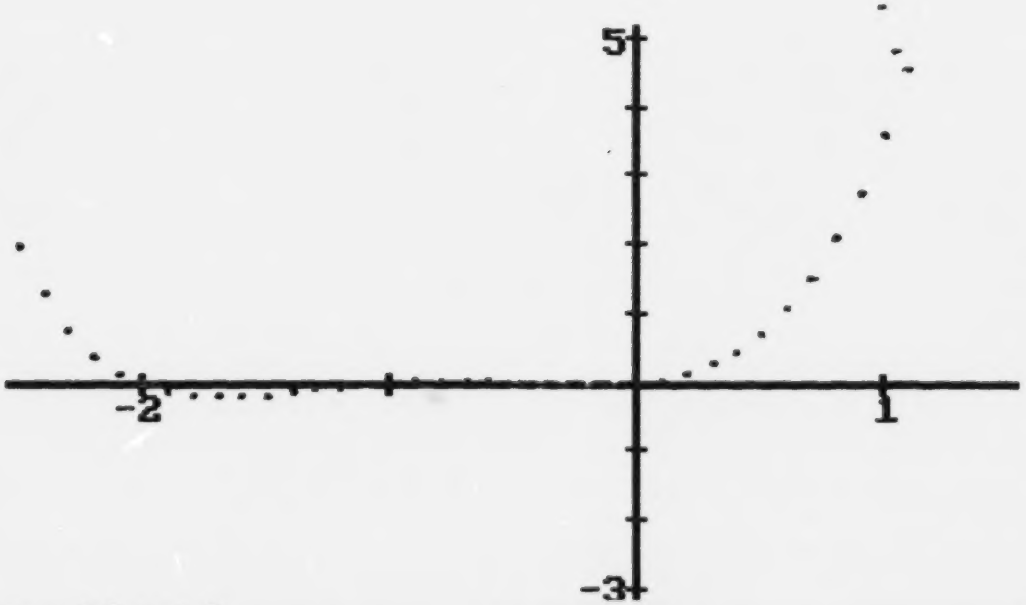
y=(x-2)*(x+1)*(x+4)

$$y=\frac{2*x^4+7*x^3+7*x^2+2*x}{x^3-x+50}$$



x: -7 to 7 y: -15 to 15

M T X Y A N E V G L W S I P D Q OR



x: -2.5 to 1.5 y: -.3 to .5

M T X Y A N E V G L W S I P D Q OR ? _

CHAPTER 2
STATDISK WORKSHEET:
Descriptive Statistics & Sample Generator
MAT 153 / Blass

NAME_____

APPROXIMATE COMPUTER TIME TO COMPLETE:_____

DIRECTIONS: Read the green directions sheet first. You may wish to practice the "modify option" before doing the blue worksheet. When asked if you wish to save data, always reply "No." When asked to answer a question, use complete sentences. Be clear and specific.

1. Select "Descriptive Statistics" from the Statdisk main menu. Then select the option labelled "**ENTER** from your own sample set."

   a) Enter 61, 63, 65, 68, 68, 69, 74, 81, 83, 91 and obtain the sample statistics and a histogram with 5 classes.

   Mean_____ Stand Dev_____

   Median_____ Variance_____

   Range_____ Q_1_____ Q_2_____ Q_3_____

   Histogram of a): Histogram of b):

   b) Modify the sample by adding 25 to each score. To do this, select the "**MODIFY** current sample," "**A**dd to sample," "e**X**it to menu." Obtain sample statistics and a histogram with 5 classes.

   Mean_____ Stand Dev_____

   Median_____ Variance_____

   Range_____ Q_1_____ Q_2_____ Q_3_____

   c) Compare your results. Observe the effect of adding a constant to all data. Generalize these results by completing the following: "If the same constant is added to (or subtracted from) each score in a data set, the effect on the mean median, standard deviation, variance, range, and histogram, is:

   _____

   _____

   _____

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2. A student works part time for a moving and rental company in Oakland County, Michigan. He collected the following load weights (in pounds) for 14 consecutive customers renting the same size trucks:

4035	4053	4046	4038	4037	4038	4029
4024	4039	4022	4050	4051	4037	4026

Based on your conclusion from Problem 1, what would be your approach to more easily find sample statistics for this set of data? _____

_____

Use your approach to obtain the sample statistics and a histogram with 5 classes.

Mean_____ Sketch histogram:

Median_____

Range_____

Stand Dev_____

Variance_____

Q_1_____ Q_2_____ Q_3_____

3. a) Select "Descriptive Statistics" from the Statdisk main menu. Then enter 23, 27, 29, 29, 32, 35, 36, 36 39, 42, and obtain the sample statistics and a histogram with 5 classes.

Mean_____ Stand Dev_____

Median_____ Variance_____

Range_____ Q_1_____ Q_2_____ Q_3_____

Histogram of a): Histogram of b):

b) Modify the sample by multiplying 15 on each score. To do this, select the "MODIFY current sample," "Multiply sample," "eXit to menu." Obtain sample statistics and a histogram with 5 classes.

Mean_____ Stand Dev_____

Median_____ Variance_____

Range_____ Q_1_____ Q_2_____ Q_3_____

c) Compare your results. Observe the effect of multiplying a constant on all data. Generalize these results by completing the following: "If the same constant is multiplied (or divided) on each score in a data set, the effect on the mean, median, standard deviation, variance, range, and histogram is that:

_____

_____

_____

4. A company lists the salaries of 15 of its classified employees (to the nearest one thousand dollars) as:

22,000	25,000	24,000	26,000	21,000
21,000	27,000	22,000	25,000	28,000
24,000	26,000	25,000	24,000	25,000

Based on your conclusion from Problem 3, what would be your approach to more easily find sample statistics for this set of data? _____

_____

Use your approach to obtain the sample statistics and a histogram with 5 classes:

Mean_____ Stand Dev_____

Median_____ Variance_____

Range_____ Q_1_____ Q_2_____ Q_3_____

Sketch histogram:

5. Refer to the student data set (see the yellow sheet). Enter the pulse rates for the 48 males. Complete the following:

Mean_____ Maximum_____ Q_1_____

Median_____ Stand Dev_____ Q_2_____

Range_____ Variance_____ Q_3_____

Does Q_2 exactly equal the median? _____ Is this what you expected? Why or Why not?

_____

Does Q_2 exactly equal the average of Q_1 and Q_3? _____ Is this what you expected?

Why or why not?_____

Sketch the histogram with 5 classes, including class marks and frequencies:

6. Calculate all statistics for the following two sets of ages using "Descriptive Statistics." Compare the results and form conclusions.

Male ages:	43	44	41	38	37	36	35	47	47	47
Female ages:	28	24	14	32	37	55	47	47	63	70

	Mean	Median	Range	Stand Dev	Variance	Q_1	Q_2	Q_3
Male:								
Female:								

Which statistics are similar?_____

_____

Which statistics are different?_____

_____

Are these results what you expected? Why or why not?_____

_____

Sketch the histograms below. How are they similar or different?_____

_____

Histogram
Males:

Histogram
Females:

7. You will investigate the effect of increasing the sample sizes. Use "Have Statdisk **CREATE** a new sample" to generate **NORMAL** samples with no digits to the right of the decimal, mean of 100, and standard deviation of 15. Obtain histograms with 8 classes of each sample. Complete the chart and histograms below.

	n = 10	n = 20	n = 100	n = 200
Min	_____	_____	_____	_____
Max	_____	_____	_____	_____
Mean	_____	_____	_____	_____
Stand Dev	_____	_____	_____	_____

As the sample size increases, what happens to the min, max, mean and standard deviation?_____

_____

Are these results what you expected? Why or why not?_____

_____

Histograms:

n = 10 n = 20 n = 100 n = 200

As the sample size increases, what happens to the histograms? _____

_____

_____

8. An outlier is an extreme value in the data set.

a) Generate a sample of 40 normally distributed values with mean of 50 and standard deviation of 5. Sketch the histogram with 10 classes and find the statistics.

Range_____ Histogram:

Mean_____

Median_____

Stand Dev_____

b) Now select "**MODIFY** current sample (Ranked)," "Replace a number," and replace the second score with 150. Sketch the histogram with 10 classes and find the statistics:

Range_____ Histogram:

Mean_____

Median_____

Stand Dev_____

c) Compare your results. By introducing an outlier, what happens to the range, mean, median, standard deviation, and histogram?_____

_____

_____

_____

9. By using statdisk and the **MODIFY** option, you can quickly convert sample scores to z scores. Since $z = \frac{x - \bar{x}}{s}$, we must Add $(-\bar{x})$ to all scores and Multiply this result by the decimal equivalent to (1/s). Enter these student exam scores using 2 places to the right of the decimal:

68, 71, 77, 80, 84, 93, 87, 94, 90, 84, 84, 66, 71, 73, 78

Record each: Mean_____ Stand Dev_____

Now modify each score to find the z scores. List all z scores corresponding to the data (in the same order as listed above).

_____

_____

Which data have z scores more than 1 standard deviation away from the mean?

_____

STUDENT DATA
(92 students taking statistics)

Notes on data:
Sex: 0 represents female; 1 represents male.
Heights are in inches.
Weights are in pounds.
Pulse rates are in beats per minute.
Siblings: Number of brothers plus the number of sisters.
Random: Subjects were instructed to choose a three digit random number.
SSN: Subjects were instructed to write the last three digits of their social security numbers.
Hand: 0 represents left-handed; 1 represents right-handed.

Subj.	Sex	Ht.	Wt.	Pulse rate	Siblings	Random	SSN	Hand
1	0	66	115	34	2	213	001	1
2	1	66	128	82	5	169	224	1
3	1	70	150	74	2	812	212	1
4	0	65	107	65	4	125	097	1
5	1	71	160	77	3	749	900	1
6	1	69	140	62	5	137	327	1
7	0	64	110	70	2	202	406	1
8	0	64	128	52	2	344	551	1
9	0	63	130	72	2	496	126	1
10	0	66	104	80	3	348	091	1
11	1	73	163	78	0	714	523	1
12	1	70	155	58	2	765	103	1
13	0	65.5	120	90	1	831	541	1
14	1	69	175	85	5	491	066	1
15	1	67	145	74	2	169	174	1
16	0	68	125	67	2	312	366	0
17	1	73	145	66	3	263	508	1
18	1	62	180	71	3	192	392	1
19	0	62	125	62	2	584	361	1
20	0	63	140	85	3	968	045	1
21	0	63	189	80	9	377	621	1
22	1	71	170	58	4	403	436	1
23	0	64	112	99	3	372	854	1
24	1	68	154	80	2	123	032	1
25	1	71	185	65	4	493	002	1
26	0	71.75	150	76	3	894	281	1
27	0	51.5	120	75	3	016	452	1
28	1	72	180	60	0	682	762	1
29	0	62	110	88	1	390	776	1
30	1	71	155	54	1	123	669	1
31	0	66	140	92	1	325	327	1
32	0	64	115	76	3	734	635	1
33	0	62	104	80	3	316	506	1
34	1	67	147	75	1	357	028	0
35	1	66	190	71	6	945	739	1
36	0	66	140	92	2	208	459	1

37	0	63	114	40	1	115	721	1
38	0	68	135	69	2	776	675	1
39	0	67	130	66	4	143	939	1
40	1	72	155	74	1	628	867	0
41	1	70	170	73	1	479	348	1
42	1	71	152	82	2	316	585	1
43	0	63	138	85	2	229	177	1
44	1	70	150	84	1	781	105	1
45	0	66.5	116	78	2	628	829	1
46	0	66	120	87	2	356	772	1
47	0	66	138	92	4	195	894	0
48	1	73	150	78	3	199	635	1
49	1	71	190	92	3	223	783	1
50	1	72	115	73	1	114	965	0
51	0	66.5	120	72	3	264	072	1
52	0	66	140	48	1	308	198	1
53	0	67	110	75	8	105	605	1
54	0	61.75	98	72	4	357	205	1
55	1	66	155	57	6	333	632	1
56	1	70	140	100	0.5	421	975	1
57	1	72	200	71	1	107	971	1
58	0	70	145	68	1	311	609	1
59	0	64	127	79	1	458	106	1
60	1	72	145	80	1	007	077	1
61	1	70	165	60	1	323	683	1
62	1	68	200	60	3	487	201	1
63	1	73	185	60	0	598	010	1
64	0	66	117	10	3	475	207	1
65	1	70	148	55	2	145	409	1
66	0	61	118	72	8	999	128	1
67	0	63	115	75	5	635	914	1
68	1	70	170	60	1	350	742	1
69	1	72	158	78	5	732	965	1
70	1	72	170	61	2	328	768	1
71	1	71	145	73	3	433	916	1
72	0	62	125	60	2	969	507	1
73	1	71	150	68	3	452	688	1
74	0	62	130	50	3	333	365	1
75	1	73	180	68	5	312	863	0
76	1	73.5	135	72	1	392	397	1
77	0	61	125	70	2	968	666	1
78	1	71	190	68	2	888	071	1
79	0	63	115	90	2	357	394	1
80	1	73	190	70	2	397	244	1
81	0	61	98	90	1	033	923	1
82	0	64	120	90	1	123	008	1
83	1	73	160	70	2	735	818	1
84	1	67	135	75	0	777	231	1
85	1	68	135	70	1	984	345	0
86	0	61	110	65	1	222	890	1
87	1	72	164	65	3	285	251	1
88	1	70	180	80	2	729	757	0
89	0	67.5	140	96	1	276	057	0
90	1	70	145	65	2	696	586	1
91	1	69	190	76	4	323	381	1
92	0	64	100	84	2	357	416	1

STATDISK

GETTING ON & OFF THE IBM PC

THE DPR LAB IS LOCATED IN D101, ON THE FIRST FLOOR OF THE D WING. HOURS ARE POSTED OUTSIDE THE LAB. YOU WILL BE USING THE IBM PC. NOTE THAT THE LAB PARAPROFESSIONALS WILL NOT BE AVAILABLE DURING LUNCH AND DINNER HOURS.

EACH UNIT ASSIGNMENT WILL BE DUE THE DAY OF THE EXAM. THESE WILL BE COLLECTED, GRADED AND RETURNED TO YOU. AFTER REVIEWING THEM, YOU WILL TURN THEM BACK IN SO THAT A FILE OF YOUR ASSIGNMENTS CAN BE KEPT.

ON:

ASK THE DPR LAB PARAPROFESSIONAL FOR STATDISK AND SIGN IT OUT. USING THE IBM PC, TURN ON THE COMPUTER. THEN INSERT STATDISK IN THE A DRIVE. WAIT UNTIL THE SCREEN FILLS WITH

AND THEN A FULL WRITTEN INFORMATION SCREEN WILL APPEAR. PRESS ANY KEY TO REACH THE MAIN MENU.

OFF:

THERE ARE TWO WAYS:

1) FROM THE MAIN MENU, SELECT "EXIT STATDISK".

2) FROM AN OPTION, YOU WILL SEE "PRESS Q TO QUIT STATDISK".

EITHER WAY YOU WILL GET TO A AND MAY REMOVE THE DISKETTE. THEN POWER DOWN THE COMPUTER. RETURN STATDISK TO THE DPR LAB DESK AND SIGN OUT.

HINT: ALWAYS READ THE GREEN INFORMATION SHEET BEFORE GOING TO THE LAB!

Statistics Display For TWOONE

Sample size = 10

Minimum ..... =	61	Sum of scores =	723
Maximum ..... =	91	Sum of squares =	53130

MEASURES OF CENTRAL TENDENCY

Mean ..... =	72.3	Geom. mean ... =	71.7
Median ..... =	68.5	Harm. mean ... =	71.2
Midrange ..... =	76.0	Quad. mean ... =	72.9

MEASURES OF DISPERSION

Samp. st. dev. =	9.8	Samp. variance =	95.3
Pop. st. dev. =	9.3	Pop. variance =	85.8
Range ..... =	30.0	Standard error =	3.1

MEASURES OF POSITION - Quartiles

Q1 = 65.0 Q2 = 68.5 Q3 = 81.0

1)	61
2)	63
3)	65
4)	68
5)	68
6)	69
7)	74
8)	81
9)	83
10)	91

Press any key to continue (ESC for Main Menu)

Statistics Display For TWOONE

Sample size = 10

Minimum ..... =	86	Sum of scores =	973
Maximum ..... =	116	Sum of squares =	95530

MEASURES OF CENTRAL TENDENCY

Mean ..... =	97.3	Geom. mean ... =	96.9
Median ..... =	93.5	Harm. mean ... =	96.5
Midrange ..... =	101.0	Quad. mean ... =	97.7

MEASURES OF DISPERSION

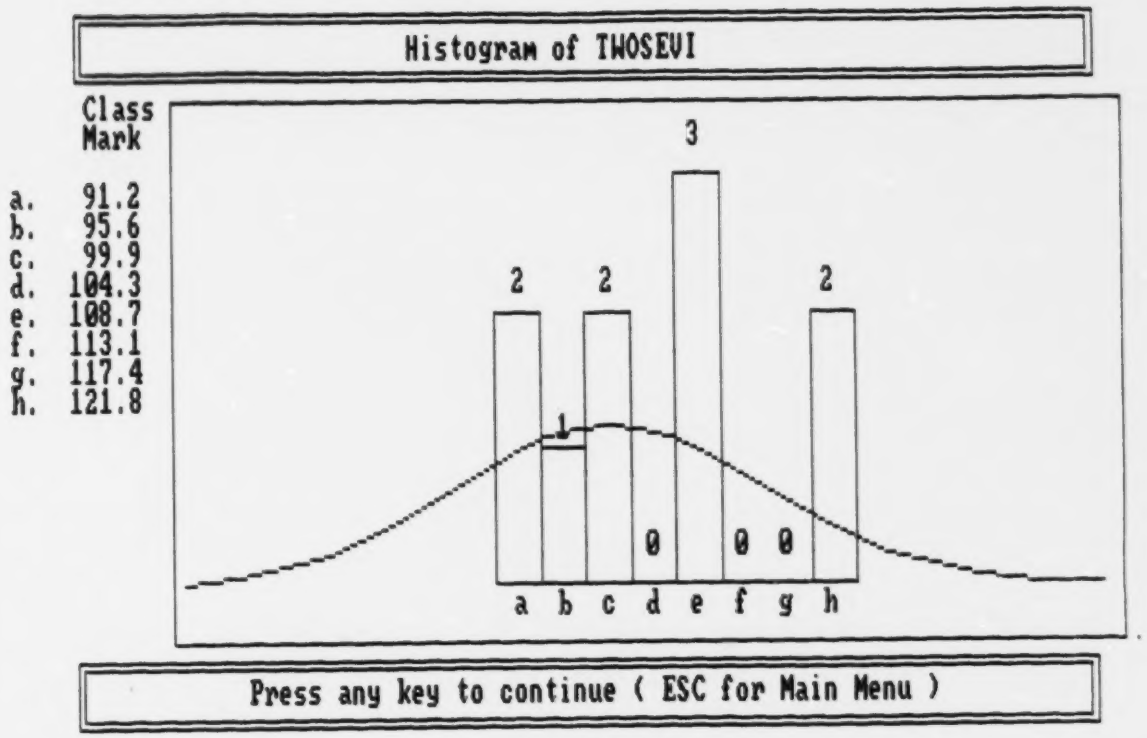
Samp. st. dev. =	9.8	Samp. variance =	95.3
Pop. st. dev. =	9.3	Pop. variance =	85.8
Range ..... =	30.0	Standard error =	3.1

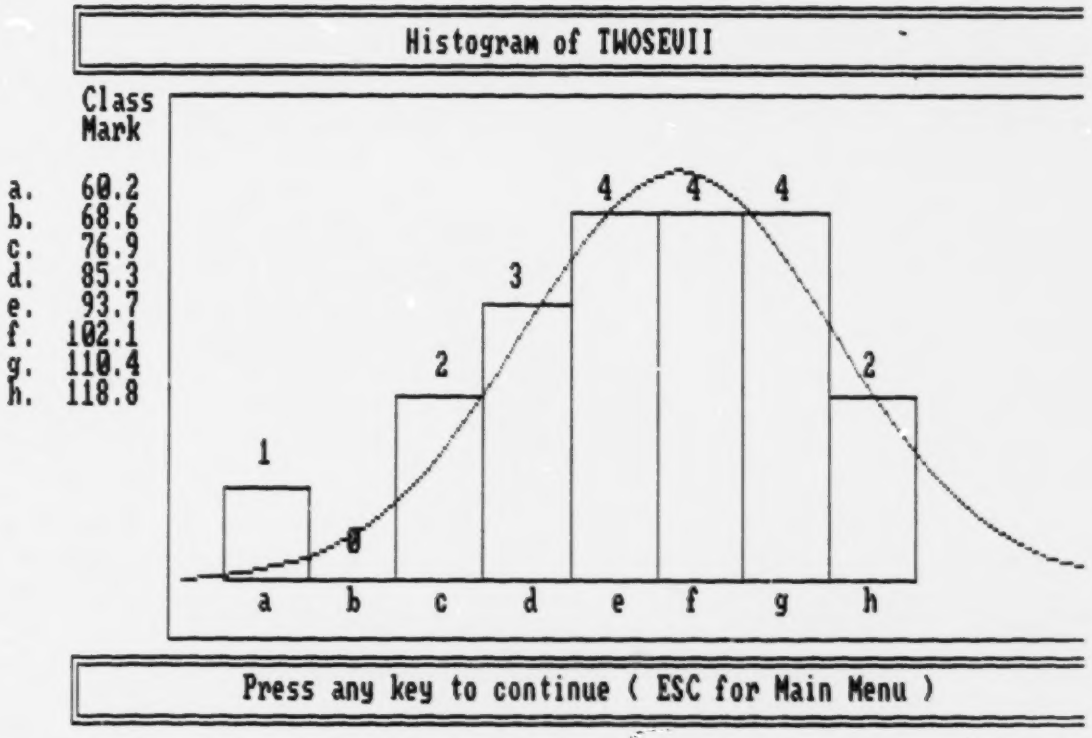
MEASURES OF POSITION - Quartiles

Q1 = 90.0 Q2 = 93.5 Q3 = 106.0

1)	86
2)	88
3)	90
4)	93
5)	93
6)	94
7)	99
8)	106
9)	108
10)	116

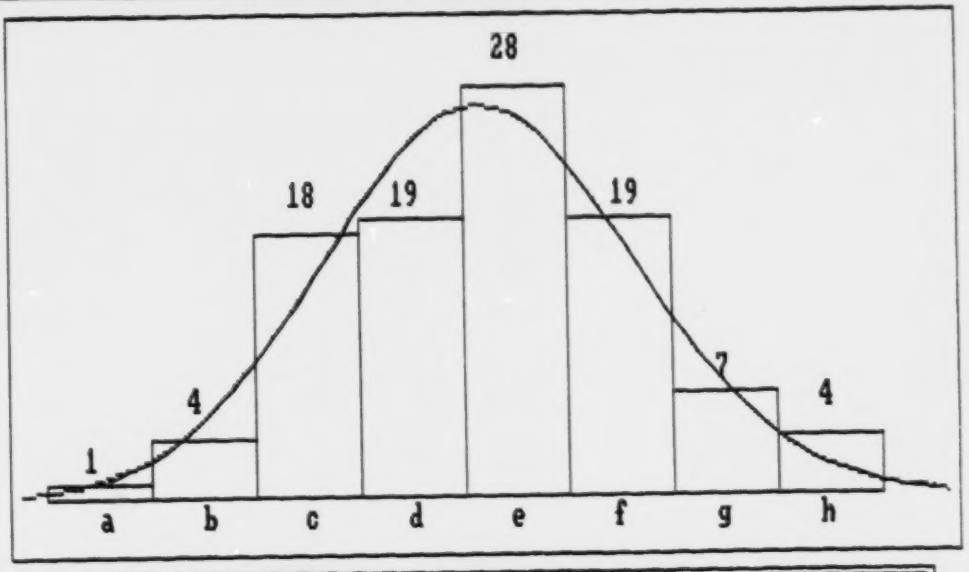
Press any key to continue (ESC for Main Menu)


Histogram of TWOSEVI
Class Mark
a. 91.2
b. 95.6
c. 99.9
d. 104.3
e. 108.7
f. 113.1
g. 117.4
h. 121.8
3
2 2 2
1
0 0 0
a b c d e f g h
Press any key to continue (ESC for Main Menu)


Histogram of TWOSEVII
Class Mark
a. 60.2
b. 68.6
c. 76.9
d. 85.3
e. 93.7
f. 102.1
g. 110.4
h. 118.8
4 4 4
3
2 2
1
0
a b c d e f g h
Press any key to continue (ESC for Main Menu)

Histogram of TWOSEVIV

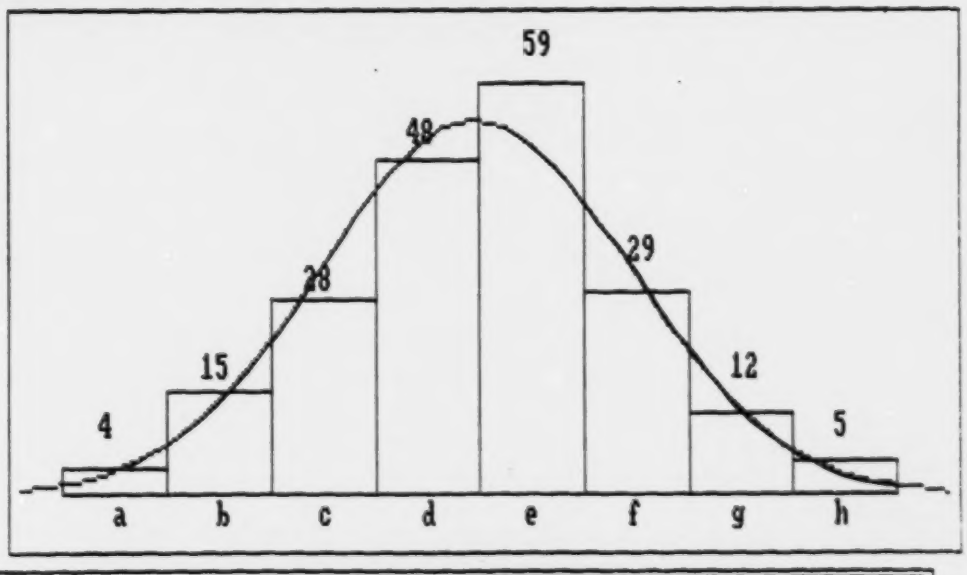
	Class Mark
a.	63.1
b.	73.2
c.	83.3
d.	93.4
e.	103.6
f.	113.7
g.	123.8
h.	133.9



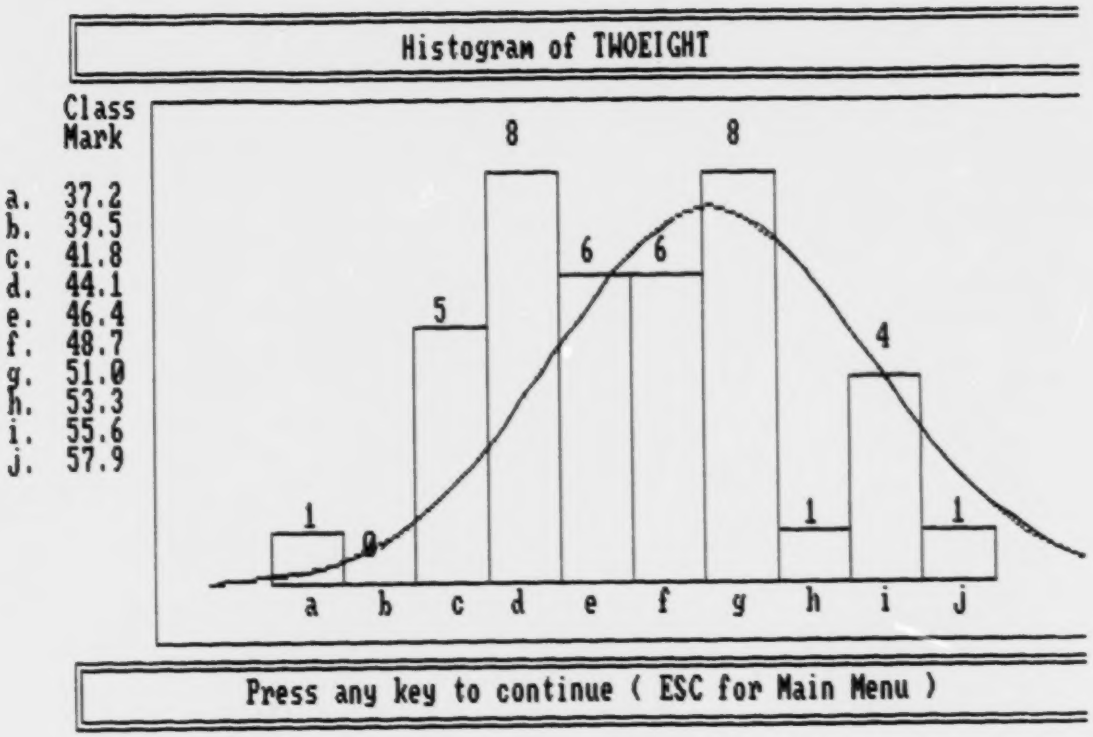
Press any key to continue (ESC for Main Menu)

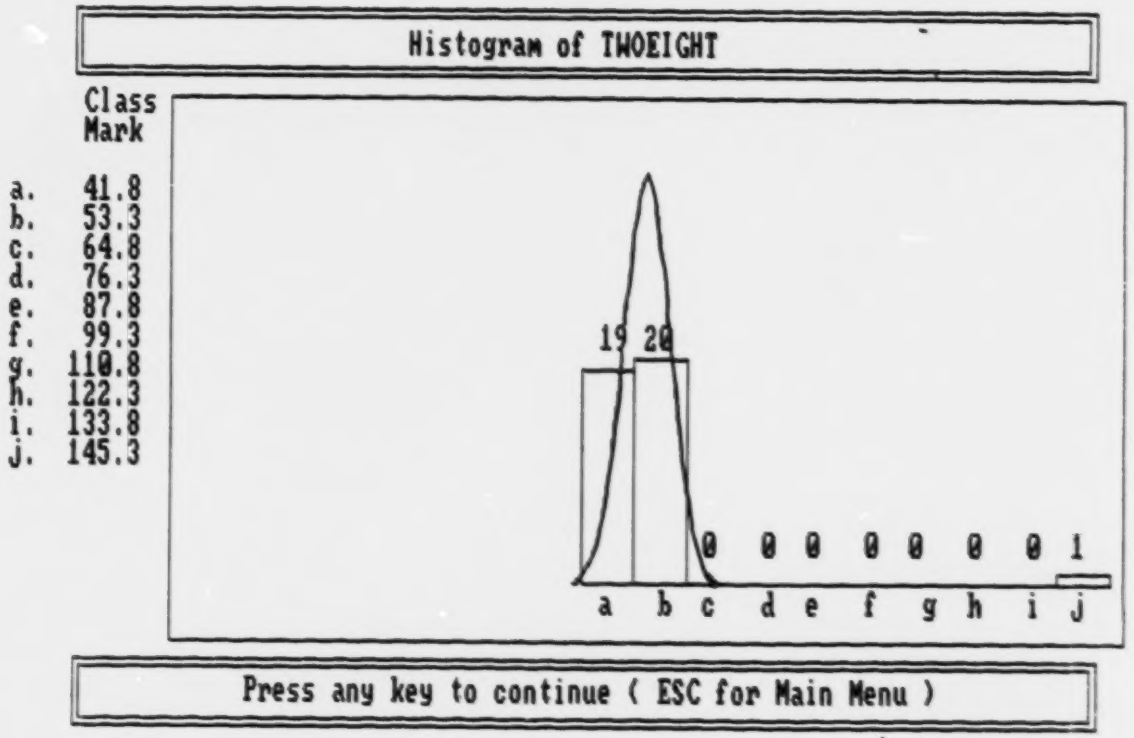
Histogram of TWOSEVV

	Class Mark
a.	65.1
b.	75.2
c.	85.3
d.	95.4
e.	105.6
f.	115.7
g.	125.8
h.	135.9



Press any key to continue (ESC for Main Menu)


Histogram of TWOEIGHT
Class Mark
a. 37.2
b. 39.5
c. 41.8
d. 44.1
e. 46.4
f. 48.7
g. 51.0
h. 53.3
i. 55.6
j. 57.9
1
0
5
8
6
6
8
1
4
1
a b c d e f g h i j
Press any key to continue (ESC for Main Menu)


Histogram of TWOEIGHT
Class Mark
a. 41.8
b. 53.3
c. 64.8
d. 76.3
e. 87.8
f. 99.3
g. 110.8
h. 122.3
i. 133.8
j. 145.3
19
20
0 0 0 0 0 0 0 1
a b c d e f g h i j
Press any key to continue (ESC for Main Menu)

SEMESTER _____ NAME _____

STATDISK CRITIQUE

SPECIFIC PROBLEMS

Refer to your personal notes on each assignment: Chapters 2,3,5,8, & 10. When answering each question, refer to specific problem numbers and be complete with your comments. Use back or attach additional pages if needed.

I. List specific problems in which the directions weren't clear.

CH. 2:

CH. 3:

CH. 5:

CH. 8:

CH. 10:

Now refer to each and suggest how the directions could be changed to make the question more understandable:

II. List specific problems which were very time-consuming on the computer.

CH 2:

CH 3:

CH 5:

CH 8:

CH 10:

Now refer to each and suggest how the directions could be changed to make the question less time-consuming:

III. List problems that seemed repetitious or that did not increase your knowledge of the material in that chapter. Include comments and explanations.

IV. List problems which significantly increased your knowledge of the material in that chapter. Include comments and explanations.

Overall Package

List an overall rating of 1 to 10, where 10 is highest, for each aspect of the STATDISK listed below. Then explain your rating decision with commments below.

RATING

_____ Amount of time needed to complete assignments on the computer.

Comments:

_____ Amount of time needed to complete assignments outside of the computer

Comments:

_____ Computer "friendliness" of STATDISK.

Comments:

_____ Clarity of assignment sheets.

Comments:

_____ Benefit of STATDISK to your understanding of homework questions in textbook.

Comments:

_____ Overall rating of STATDISK.

Comments:

Suggestions for Inclusion of STATDISK in Winter Semester

What are your suggestions? Should it be continued? Should it be required to optional? Should it be used to replace an exam? A final? As a project that could be used to improve your course average? Any ideas? If you feel it was not a valuable software package, what would have improved it?

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STUDENT CRITIQUES

After doing Statdisk I could really understand what was happening. I always have to know why something happens, I just can't say, "Well, the answer is right," and go on to the next problem. I really enjoyed working with Statdisk. It shows you exactly what happens with the graphs, etc. Keep using it!

I feel it was beneficial to the interpretation of what bottom line numbers represented. Some of them were thinkers in making comparisons when changes were made with data. It's a nice option to be offered.

It helped more for an overall understanding of how everything fits together. Helped to put Stats in perspective in relation to the real world. Helped me a lot to feel more comfortable with computers and to understand field of research better.

It made me understand in a different way. I was able to see differences, like if you change one number, what effect did it have. You do not get to do that as easily with homework problems.

It made me understand why certain things would happen when we would compare the answers of a few different questions.

Some students who are taking Stats at OCC are preparing for a higher Stats class and I think most schools require computer use. The experience is helpful to students, even if the next class does not use such a friendly program. I think the chance to use Statdisk at OCC could do nothing but help for the future. I think that I really gained knowledge about Statistics by using this program. It is a valuable program and students should take full advantage of it.

I would highly encourage students to try Statdisk.It made the understanding of Statistics complete.

VALUE OF STUDENT COMPUTER ASSIGNMENTS

1. In the computer lab, students are active learners.

2. Students must learn to think for themselves.

3. Realistic situations motivate students.

4. Students become more flexible and greater risk-takers in problem-solving situations.

5. Discussions between students increased their verbal facility of the subject.

6. Student worksheets increased student abilities to express mathematical ideas in writing.

7. New student-teacher-technology interaction patterns imply new roles and responsibilities for students.

SOFTWARE CATALOGS FOR ALL SUBJECTS

Sunburst Communications
101 Castleton Street
Pleasantville, New York 10570-3498
1-800-628-8897

William K. Bradford Publishing Company
310 School Street
Acton, MA 01720-9917
1-800-421-2009

Paradigm Publishing International
7500 Flying Cloud Drive, Suite 655
Eden Prairie, Maine 55344
1-800-535-6865

EduSoft
P.O. Box 2560-VX
Berkeley, CA 94702-9983
1-800-EDUSOFT

SHAREWARE CATALOGS FOR ALL SUBJECTS

CWI Information Services
P.O. Box 4851
Anaheim, CA 92803
1-800-777-5636

Data Outlet Shareware
P.O. Box 776
Macon, Georgia 31202-0776
1-800-347-4306

Sizzleware Shareware Library
P.O. Box 6429
Lake Charles, LA 70606
1-800-356-2697

MATHEMATICS SOFTWARE

Cactusplot Company
4712 E. Osborn
Phoenix, Arizona 85018
1-602-840-3811

Derive
MathWare
Urbana, IL 61801
1-217-384-3196

Mathematics, Statistics & Logic
INTELLIMATION-Library for MacIntosh
1-800-3-INTELL

STAT-STAR
Academy Software, Inc.
4139 Daventry Lane
Palm Harbor, Florida 34685
1-800-541-0292

The Math Lab
10893 Leavesley Place
Cupertino, CA 95014
1-408-252-3493

Functioning Graphing
Kamischke Software
1220 Reads Run
Traverse City, MI 49684

MINITAB, Inc.
3081 Enterprise Drive
State College, PA 16801
1-814-238-3280

SYSTAT, Inc.
1800 Sherman Avenue
Evanston, IL 60201
1-708-864-5670

Computer Activities
Houghton Mifflin Co.
Two Park Street
Boston, MA 02108

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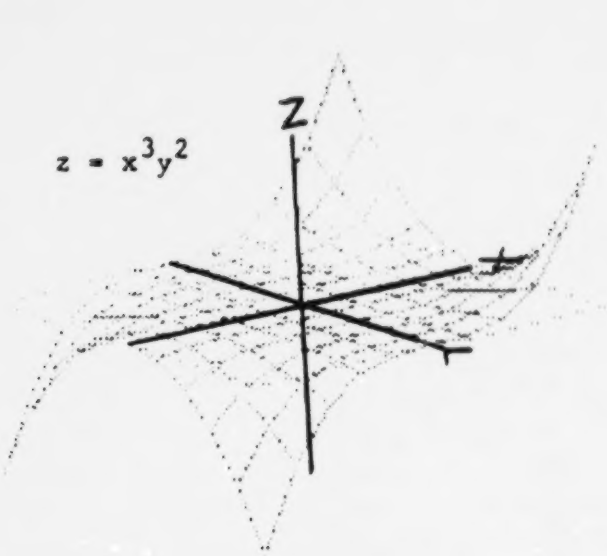
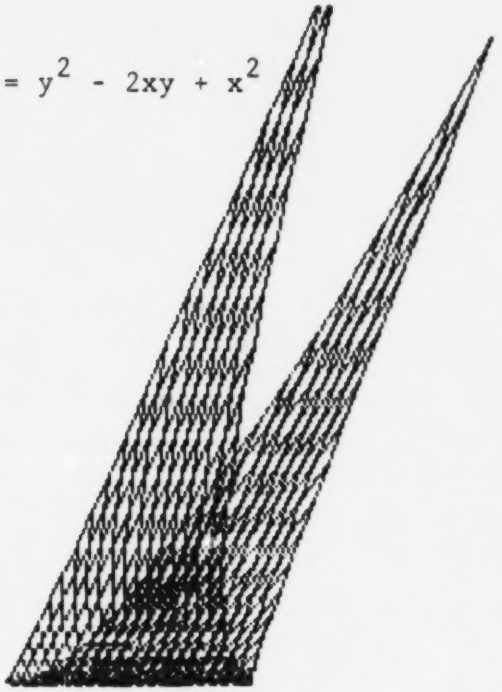
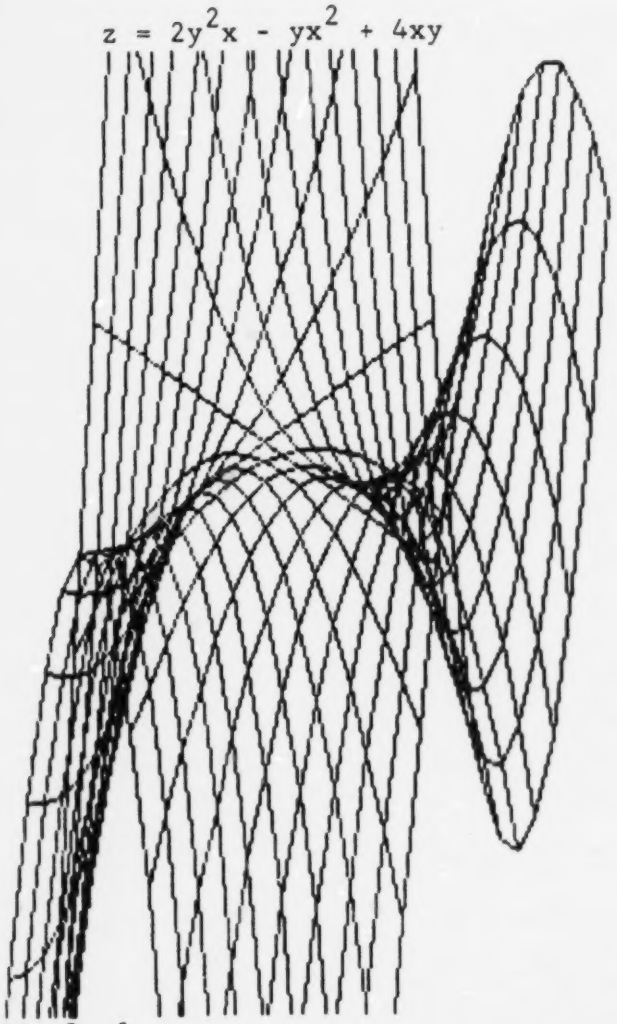
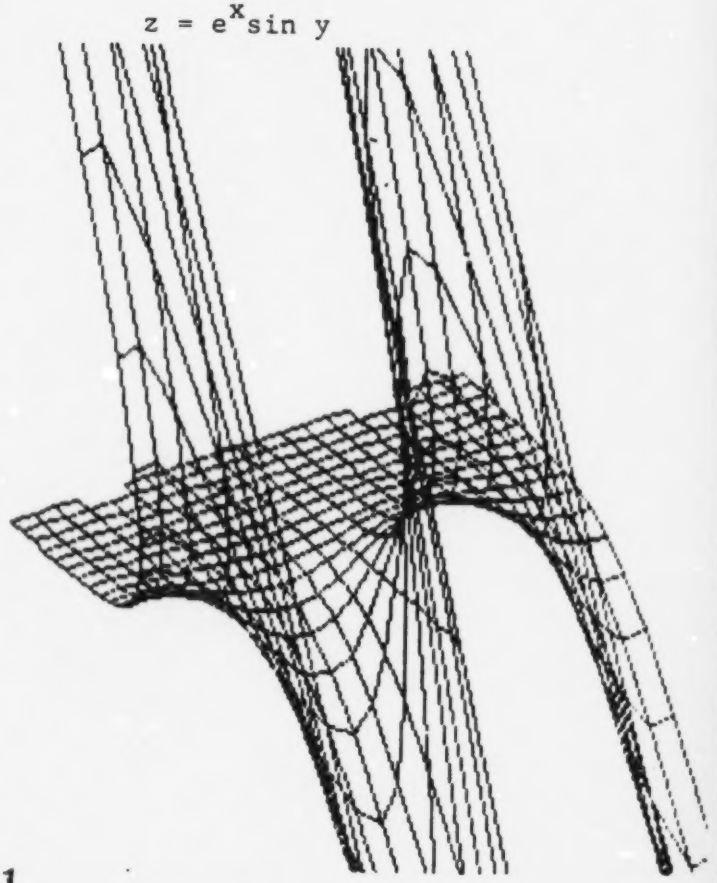
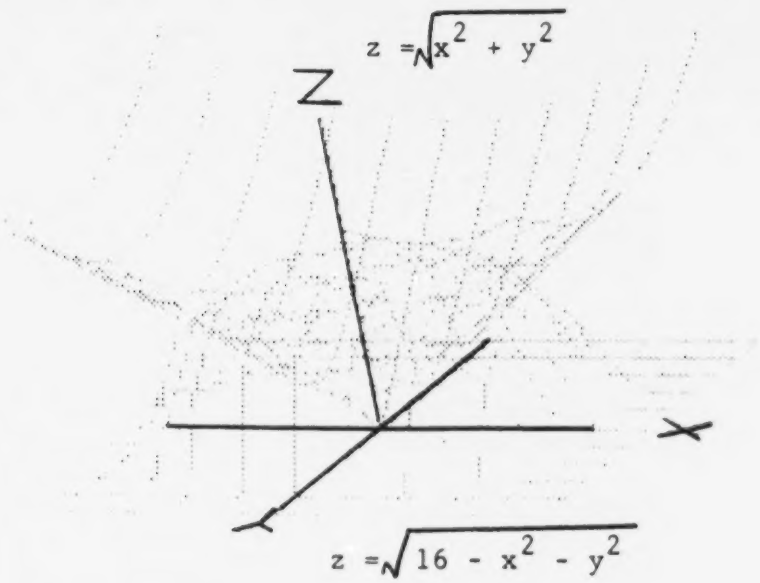
THREE DIMENSIONAL SURFACES










$z = \sqrt{x^2 + y^2}$
Z
X
Y
$z = \sqrt{16 - x^2 - y^2}$

DIRECTION FIELDS

1) To determine if $y'=f(x,y)$ has a solution through (x_0,y_0):
   - sketch the direction field associated with $y'=f(x,y)$.
2) $f(x,y)=c$ determines the isoclines, or curves of constant inclination.
   - slope = $y' = f(x,y) = c$
   - every solution curve passing through a particular isocline does so with the same slope.
3) The direction field is the totality of short line segments that have midpoints on the isoclines and that have slopes equal to the value of c.
4) A plotted sequence of these lineal segments suggests the shape of a solution curve passing through (x_0,y_0).
5) A direction field can give only a crude form of an approximation to a numerical value of the solution of the initial-value problem when x is close to x_0.
6) Each point in the plane gives rise to a different solution.
7) Solutions depend on the initial conditions.

$$y' = x^2 + y^2 \quad (1)$$

FIGURE 2-1 Direction Field

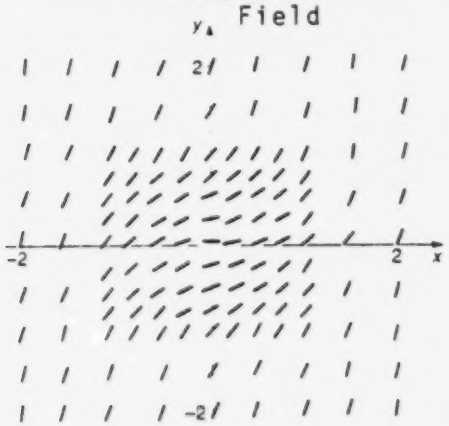
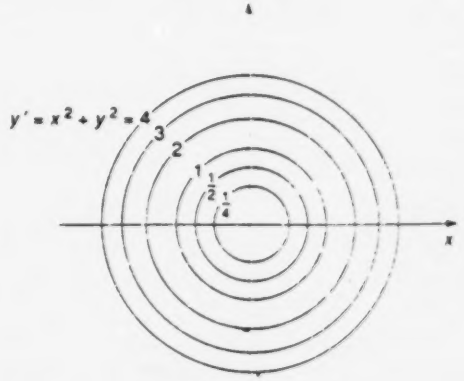


FIGURE 2-2 Isoclines



Starting at any point in the plane, we attempt to sketch a curve whose tangent lines at every point agree with the direction field. Figures 2–3 and 2–4 show the results of such an attempt, starting at the origin.

FIGURE 2-3

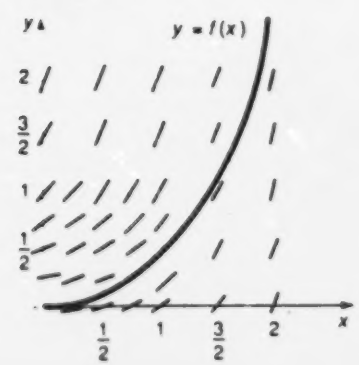
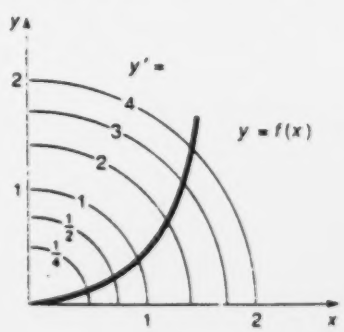
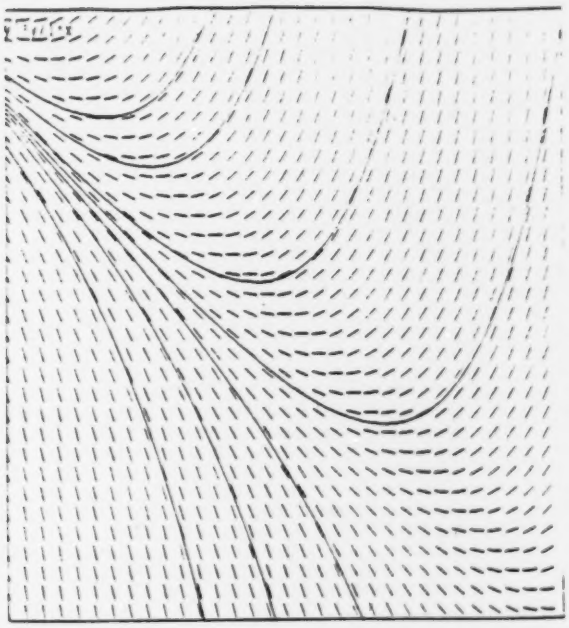
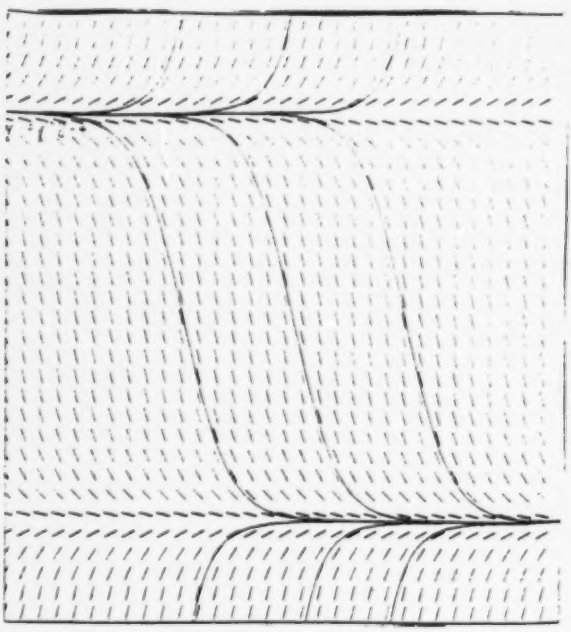


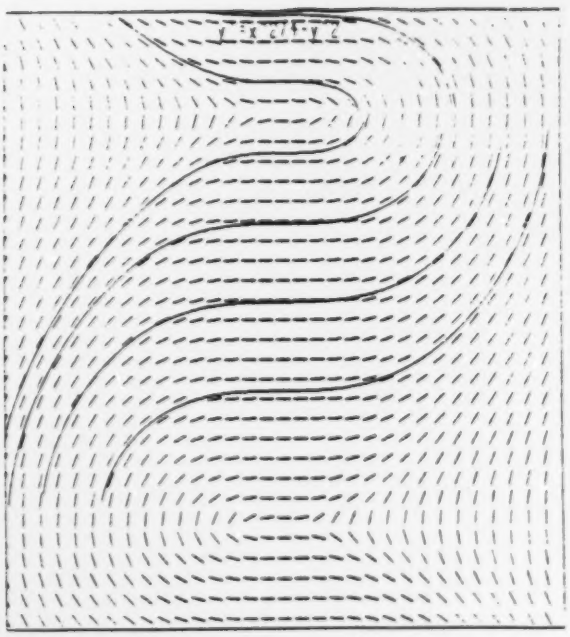
FIGURE 2-4

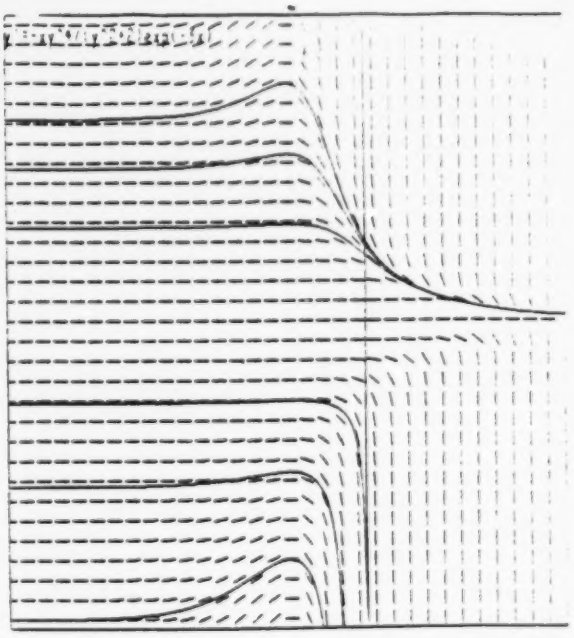


The curve approximately represents one solution $y = f(x)$ to equation (1). This is a solution satisfying $f(0) = 0$, since the curve passes through (0,0). Other solutions, passing through any given point in the xy-plane, can be approximated in the same way. (Try it.) In Figures 2–3 and 2–4, the value $f(1)$ appears to be somewhere between 1/4 and 1/2, and apparently $f(2) > 2$.







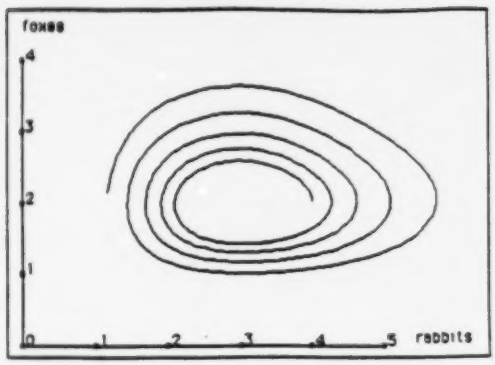


Predator And Prey

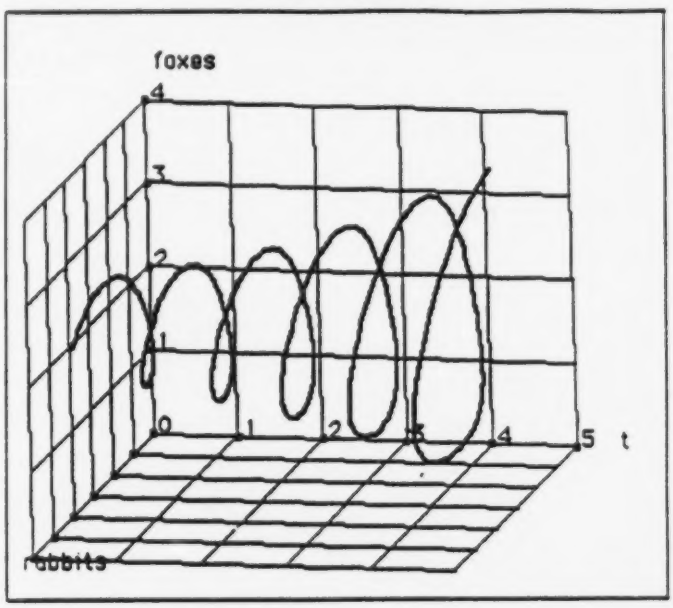
Creating the Model

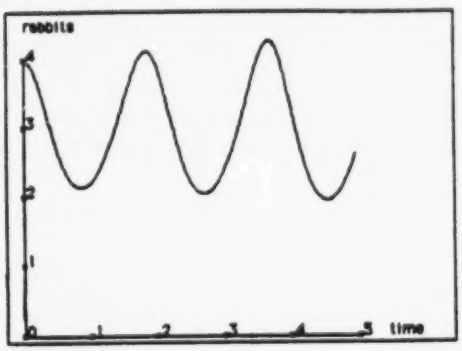
t	0	<0>	t+.01
a	0.08	<0.08>	.08
b	0.04	<0.04>	.04
c	0.02	<0.02>	.02
d	0.06	<0.06>	.06
rabbitBirths	0	<0>	a*rabbits
rabbitDeaths	0	<0>	b*rabbits*foxes
foxBirths	0	<0>	c*rabbits*foxes
foxDeaths	0	<0>	d*foxes
changeInRabbits	0	<0>	rabbitBirths-rabbitDeaths
changeInFoxes	0	<0>	foxBirths-foxDeaths
rabbits	4	<4>	rabbits+changeInRabbits
foxes	2	<2>	foxes+changeInFoxes

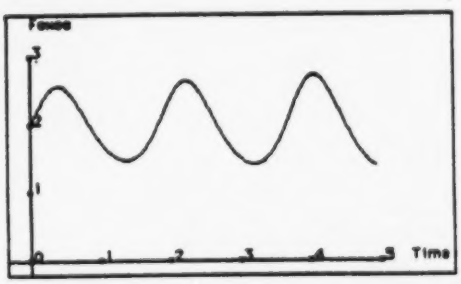
Phase Portrait

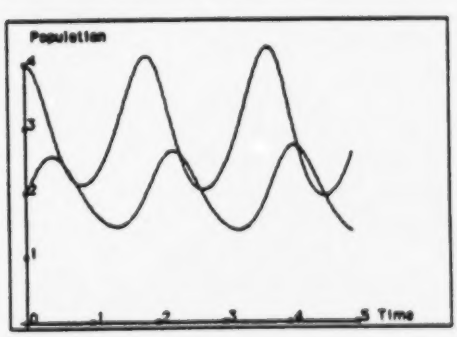


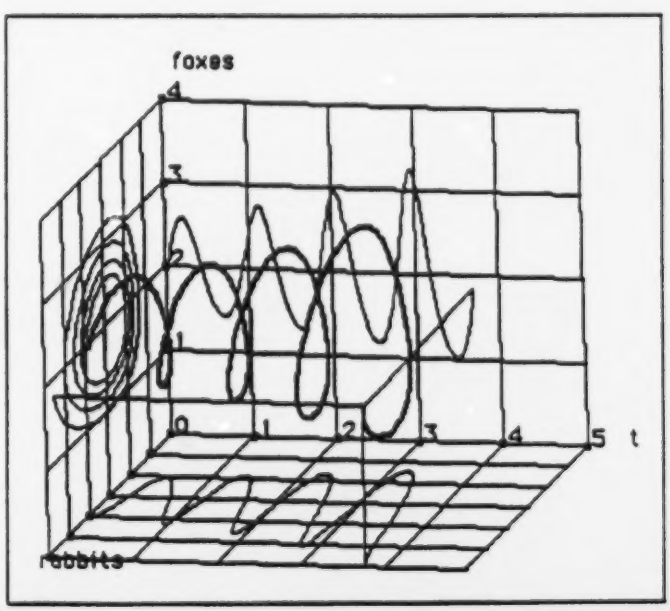
Third Dimension











WORKSHEET
LIMITS

NAME_____

1. Given $f(x) = x^2$, fill in the following charts for the values of x given.

x	f(x)
2.5	
2.2	
2.1	
2.01	
2.001	
2.0001	
2.00001	

x	f(x)
1.5	
1.8	
1.9	
1.99	
1.999	
1.9999	
1.99999	

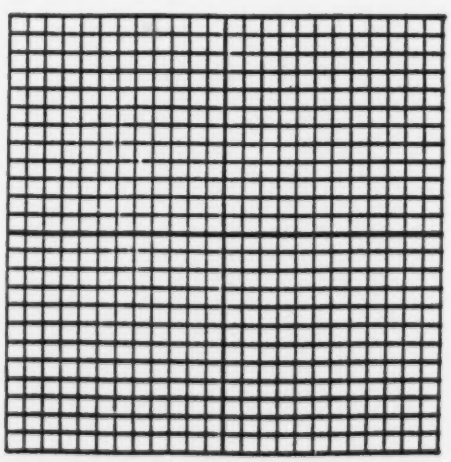
It appears that as x comes closer and closer to 2 (approaches 2), f(x) comes closer and closer to _____ (approaches _____).

[Note that this happens when we approach 2 by using numbers smaller than 2 and when we approach 2 using numbers larger than 2; i.e., we approach 2 from the left and from the right.]

This is said to be the <u>limit</u> (<u>lim</u>) of f(x) as f(x) approaches 2 ($x \to 2$).

Mathematically we write: [$\lim_{x \to 2} f(x) = 4$; or $\lim_{x \to 2} (x^2) = 4$]

Confirm your conclusions by graphing $y = x^2$ and observing what is happening to y as x gets closer to 2 from the right and from the left.



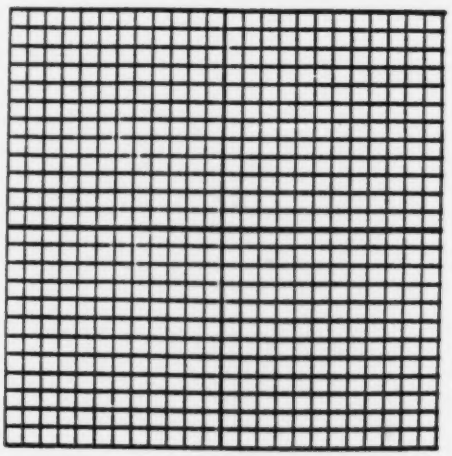
2. Given $f(x) = \frac{\sin x}{x}$, fill in the chart. (Values are in radians.)

x	f(x)
1.0	
0.8	
0.5	
0.2	
0.1	
0.01	
0.001	
0.0001	

x	f(x)
-1.0	
-0.8	
-0.5	
-0.2	
-0.1	
-0.01	
-0.001	
-0.0001	

Based on the above data, then, the $\lim_{x \to 0} \frac{\sin x}{x} =$ _____

Graph $y = \frac{\sin x}{x}$ and see if your answer makes sense.



3. Do the same for $f(x) = \frac{|x|}{x}$

x	f(x)
1.0	
0.8	
0.5	
0.2	
0.1	
0.01	
0.001	
0.0001	

x	f(x)
-1.0	
-0.8	
-0.5	
-0.2	
-0.1	
-0.01	
-0.001	
-0.0001	

As x approaches 0 from the left ($x \to 0^-$), f(x) approaches _____.

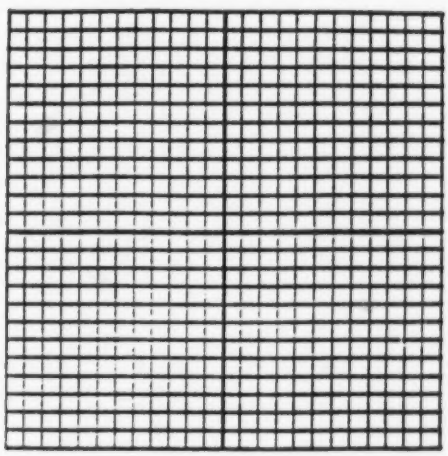
And, as x approaches 0 from the right ($x \to 0^+$), f(x) approaches _____.

Is there one single number that f(x) approaches as x approaches 0?

[Since there is not, there is no general limit for f(x) as x approaches 0.] We can however discuss and use the left limit and the right limit separately. If we do this we say:

$$\lim_{x \to 0^-} f(x) = _____ \quad \text{and} \quad \lim_{x \to 0^+} f(x) = _____ \ ]$$

Confirm the above by graphing $y = \frac{|x|}{x}$ and observing what is happpening to y as x gets closer to 0 from the right and from the left.



4. Do the same for $f(x) = \frac{1}{x}$

x	f(x)
2.0	
1.0	
0.5	
0.2	
0.1	
0.01	
0.001	
0.0001	

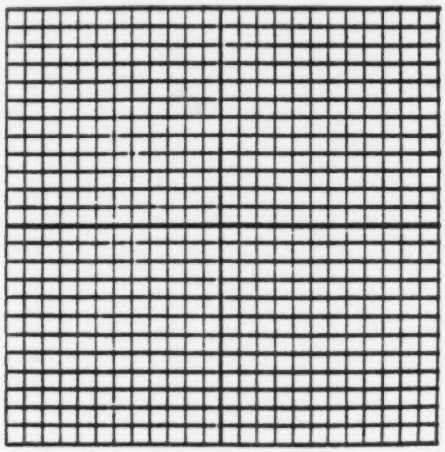
x	f(x)
-2.0	
-1.0	
-0.5	
-0.2	
-0.1	
-0.01	
-0.001	
-0.0001	

What is happening to f(x) as x approaches 0 from the left?

What is happening to f(x) as x approaches 0 from the right?

Is there a number f(x) is approaching as x approaches 0 from the left and the right? What do you think you can say about the limit of f(x) in a case like this?

Graph $y = \frac{1}{x}$ and see if your answer makes sense.



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LIMITS

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2.2	
2.1	
2.01	
2.001	
2.0001	
2.00001	

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1.5	
1.8	
1.9	
1.99	
1.999	
1.9999	
1.99999	

It appears that as x comes closer and closer to 2 (approaches 2), f(x) comes closer and closer to _____ (approaches _____).

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1.0	
0.8	
0.5	
0.2	
0.1	
0.01	
0.001	
0.0001	

x	f(x)
-1.0	
-0.8	
-0.5	
-0.2	
-0.1	
-0.01	
-0.001	
-0.0001	

Based on the above data, then, the $\lim_{x \to 0} \frac{\sin x}{x} =$ _____

3. Do the same for $f(x) = \frac{|x|}{x}$

x	f(x)
1.0	
0.8	
0.5	
0.2	
0.1	
0.01	
0.001	
0.0001	

x	f(x)
-1.0	
-0.8	
-0.5	
-0.2	
-0.1	
-0.01	
-0.001	
-0.0001	

As x approaches 0 from the left ($x \to 0^-$), f(x) approaches _____.

And, as x approaches 0 from the right ($x \to 0^+$), f(x) approaches _____.

Is there one single number that f(x) approaches as x approaches 0?

[Since there is not, there is no general limit for f(x) as x approaches 0.] We can however discuss and use the left limit and the right limit separately. If we do this we say:

$\lim_{x \to 0^-} f(x) =$ _____ and $\lim_{x \to 0^+} f(x) =$ _____]

4. Do the same for $f(x) = \frac{1}{x}$

x	f(x)
2.0	
1.0	
0.5	
0.2	
0.1	
0.01	
0.001	
0.0001	

x	f(x)
-2.0	
-1.0	
-0.5	
-0.2	
-0.1	
-0.01	
-0.001	
-0.0001	

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